Tactical Alpha: A Quantitative Case for Active Asset Allocation

Adam Butler,

Michael Philbrick

Rodrigo Gordillo

Andrew Butler

ABSTRACT

Grinold linked investment alpha and Information Ratio to the breadth of independent active bets in an investment universe with his Fundamental Law of Active Management. Breadth is often misinterpreted as the number of eligible securities in a manager's investment universe, but this ignores the impact of correlation. When correlation is considered, a small universe of uncorrelated assets may explain more than half the breadth of a large stock universe. Given low historical correlations between global asset classes in comparison with individual securities in a market, we make the case that investors may be well served by increasing allocations to Tactical Alpha strategies in pursuit of higher Information Ratios. This hypothesis is validated by a novel theoretical analysis, and bolstered by two empirical examples applied to a global asset class universe and U.S. stock portfolios.

Modern markets show considerable micro efficiency (for the reason that the minority who spot aberrations from micro efficiency can make money from those occurrences and, in doing so, they tend to wipe out any persistent inefficiencies). In no contradiction to the previous sentence, I had hypothesized considerable macro inefficiency, in the sense of long waves in the time series of aggregate indexes of security prices below and above various definitions of fundamental values.

Paul Samuelson in a private letter to Robert Shiller (2001)

Corresponding author
Email addresses: your email (Adam Butler), my email (Andrew Butler), your email (Michael Philbrick), your email (Rodrigo Gordillo)

INTRODUCTION

It has long been considered prudent investment policy to separate the asset allocation decision from the active investments in portfolios. Typically, asset allocation is expressed as a semi-permanent policy or reference portfolio guided by the board and investment committee based on intermediate or long-term estimates of risk premia across the eligible asset universe.

Once the policy portfolio weights are struck, the investment staff set about selecting managers within each of the asset class silos with the goal of harvesting alpha from security selection.

Many investors perceive that the opportunity to generate incremental excess returns is much higher in the security selection space than the asset allocation space because there are vastly more securities (i.e. stocks and bonds) than there are asset classes (i.e. stock market and bond market indexes, commodities, etc.). This perception influences the relative priority placed on the pursuit of alpha from active security selection relative to active asset allocation, or, 'Tactical Alpha'. This paper will address this imbalance and offer compelling evidence that equal priority (at least) should be placed on generating excess returns from active asset allocation, even at the expense of sacrificing active security selection.

For most institutions, the asset allocation decision and the security selection decision embody a tradeoff. This is due to the structural frictions embedded in the use of external managers employed in an attempt to beat the benchmark through active security selection in a specific market. Unfortunately, institutional investors perceive that their ability to move dynamically in and out of asset classes is constrained by the redemption policies of their traditional investment managers. As such, agile rotation among and between markets and asset classes is perceived to be difficult on shorter term horizons of, say, less than one year.

Market inefficiencies exist for a variety of reasons, such as asymmetric information, tax frictions, and emotional biases. Perhaps the most economically significant inefficiencies stem from structural constraints imposed on a large segment of investors. We view the structural bias in favour of security selection alpha versus Tactical Alpha among institutional and private investors alike as an important example of this type of inefficiency. So long as Tactical Alpha is ignored by most investors, the Tactical Alpha space is likely to remain especially ripe with anomalies. As a result, we assert that Tactical Alpha active asset allocation represents one of the most economically important sources of excess returns available to investors in public markets.

SHOULDERS OF GIANTS

Most previous studies on the impact of asset allocation relative to security selection have been performed on pension funds and mutual funds, and explore the degree to which total portfolio variance is explained by deviations from institutions' policy portfolios. The studies are structured as attribution analyses, where portfolio returns are disaggregated into returns due to the policy portfolio and active returns, which in most studies are defined as the residual not accounted for by the policy portfolio.

Brinson et al. (1986, 1991) regressed monthly portfolio total returns for pension funds against the monthly returns to each funds policy portfolio, and determined that the policy portfolio explains approximately 90% of the monthly variance in total returns. Many citations of Brinson's original publications in this field falsely suggest that their analysis makes conclusions about return attribution.

Table 1: Proportion of total return explained by policy portfolio (Ibbotson and Kaplan, 2000)

Study	Average %	Median %
Brinson 1986	112	-
Brinson 1991	101	-
Ibbotson 2000 [Mutual Funds]	104	100
Ibbotson 2000 [Pension Funds]	99	99

In reality, Brinson's study mainly proved that once an institution sets a strategic asset allocation, it tends to stick to it wtih minimal deviation through time. Any meaningful variation from policy weights would, presumably, result in a r-squared coefficient with considerably lower explanatory power.

Ibbotson and Kaplan (2000) recognized the omnipresence of misperception around Brinson's papers and set out to correct this in their paper, "Does Asset Allocation Policy Explain 40, 90, or 100% of Performance?". Aside from validating Brinson's original analysis, they answered two related questions: to what degree does asset allocation explain the variability of performance between funds and institutions, and; to what degree does asset allocation explain the level of long-term performance?

To determine how well asset allocation explained the dispersion in returns across funds, the authors performed a cross sectional regression of returns from funds and institutions against respective policy benchmarks. They determined that 40% of the difference in returns across funds is explained by differences in asset allocation policy, with the balance determined by a combination of tactical shifts, sector bets, security selection, and fees. Notably, their analysis did not yield attributions among these remaining variables. We intend to address this directly.

Lastly, Ibbotson and Kaplan performed an attribution analysis to determine the percent of long-term performance explained by asset allocation. They calculated the long-term performance of each fund's policy portfolio and compared it against actual long-term fund returns. The results of this analysis are described in Table 1.

Ibbotson and Kaplan stated that, on average, asset allocation explained about 104% of long-term returns. How might we interpret this finding? Recall that the total return to portfolios were decomposed into the total return to the fund's policy portfolio using asset class benchmarks, plus the active return, minus trading frictions. So the results of this study demonstrate that, over the periods studied in the analyses, the average institutional investor lost 4% of total return to fees, ineffective active management, or poor manager selection.

Combined with the original analysis by Brinson, who makes the strong case that institutions make very few deviations from policy weights over time, one is left to conclude that the vast majority of the dispersion and performance decay observed by Ibbotson and Kaplan was due to fees and poor active security selection. This is a troubling condemnation of traditional forms of active management in general.

Since institutions apparently do not make meaningful active bets in asset allocation, we are left to ponder how much opportunity was squandered by ignoring this portion of the decision tree. Fortunately, Assoe et al. (2006) [ALP] performed an analysis, modeled after Kritzman and Page (2003), which applied a creative approach to answer this question. ALP used a normative framework, in which the potential returns in each quarterly period from 1985 - 2005 were explored for a large set of constrained, randomly generated asset class portfolios and security portfolios.

In the analysis by ALP, benchmark weights were assigned for a theoretical fund that included cash (5%), bonds (30%), stocks (40%), real estate (10%), private equity (10%), and commodities (5%). At the start of each annual period, 100 draws were made from the asset pool, where each draw was weighted according to the above proportions, and represented 1% of the final portfolio for that year. The 100 draws of 1% allocations thus result in a fully invested portfolio. The returns to the random portfolios were then computed for each quarter of the subsequent year, after which a new random portfolio was constructed in the same way. This process was repeated 10,000 times each year for the 20 year period from 1985 through 2005, with each repetition representing one sample portfolio.

The purpose of this procedure was to generate a large sample of random portfolios produced exclusively from marginal changes to asset allocation around prescribed weights. To this end, the dispersion of portfolio returns is due exclusively to changes in asset allocation, as opposed to the other variables cited by Ibbotson and Kaplan.

A similar procedure was then used to generate stock portfolios from a long-term S&P 500 stock dataset. In this case stock portfolios were created at the start of each year by randomly selecting 100 stocks. The probability of inclusion at each random draw for any given stock was equal to the stock's current weight in the index, so that over many trials the average weight for each stock would converge to the stocks' market cap weighting, though each single random portfolio would deviate normally around these value. This procedure was also repeated 10,000 times over the entire 20-year investment period, with each repetition representing one sample portfolio.

ALP were concerned with measuring the dispersion in returns between top performing random portfolios and worst performing random portfolios over time. This dispersion would serve as a proxy for the breadth of opportunity for a manager to out- or under-perform. They measured dispersion by calculating the performance difference between the 5th and 95th percentile portfolios in each quarter for the asset allocation portfolios and the stock selection portfolios. The authors asserted that this dispersion metric proxies the breadth of bets available within each asset universe at each period. They documented three important conclusions:

1. The relative importance of asset allocation and security selection is time-dependent.
2. The asset allocation driven dispersion is more volatile than the security selection induced dispersion.
3. The security selection activity generates as much dispersion in active return as asset allocation so that it cannot be unequivocally declared that one activity is structurally more or less important than the other.

We would add a few other observations. First, the paper deliberately constrains the deviations in allocations to the six asset classes by weighting them in the asset pool according to a typical institutional weighting scheme. While this assumption is consistent with the current decision-making latitude of many institutions, it does not allow the analysis to account for the full opportunity set offered by an unconstrained asset allocation decision, such as the opportunity set available to CTAs or unconstrained asset allocators seeking Tactical Alpha.

Second, the authors do not seek to explore the cause of the time-varying nature of the relative value of asset allocation versus security selection. That is, at times the asset allocation contribution dominates the contribution of security selection, while at other times the reverse is true. We wonder, what are the driving forces behind these time-varying shifts?

STRUCTURE

This paper is organized in two Parts as follows. First, we explore the concept of Information Ratio from the perspective of Grinold's Fundamental Law of Active Management (Grinold, 1989), which decomposes the Information Ratio into two terms encapsulating the roles of skill and breadth as performance drivers. Next, to facilitate understanding of this paper by a wider audience, we provide a brief introduction to our primary analytical tool: Principal Component Analysis. Following that, we discuss a theoretical study by Staub and Singer (2011), and propose an extension of their framework to explore the proportion of total portfolio breadth attributable to Tactical Alpha versus security selection decisions under various correlation assumptions.

In Part Two we focus on empirical analysis with the goal of quantifying the proportion of breadth derived from a universe of 10 global asset classes plus cash versus the excess breadth introduced by adding first S&P 100 stocks, and then S&P 500 stocks. We employ both analytical and normative investigations. First we analytically investigate the number of statistically significant independent bets that are endogenous to each universe by extending a factor analysis technique proposed by Polakow and Gebbie (2008). We employ random matrixes to ensure statistical robustness. Finally, we perform a normative analysis of breadth by generating random portfolios of assets and stocks.

PART I: THEORY

INFORMATION RATIO AND THE FUNDAMENTAL LAW OF ACTIVE MANAGEMENT

Recall that traditional alpha is the residual return from security selection (SS) after accounting for a strategy's beta with a market index or benchmark (B).

$$\propto_{SS} = r_{SS} - r_B \times \beta_{SS}$$

Tactical Alpha is somewhat more ephemeral because there is often no obvious benchmark. In practice, Tactical Alpha is often defined as active excess return relative to a policy portfolio that is achieved through Tactical Asset Allocation decisions. Tactical Asset Allocation is simply active deviation from benchmark weights in the policy portfolio due to changing estimates for risk premia across asset classes over time.

Given that Tactical Alpha seeks to deliver excess returns by incurring active risk relative to a policy portfolio, we argue that the Information Ratio (IR) is the most appropriate tool to measure value added. Of course, IR is also used extensively to measure the value of traditional sources of active risk in the realm of security selection. This makes the IR a practical unbiased ratio for a comparison of these two sources of active risk. It should be noted that the IR and the Sharpe Ratio are equivalent when the benchmark is a risk-free asset.

$$IR = \frac{E[Rp - Rb]}{\sqrt{var[Rp - Rb]}}$$

BREADTH AND CORRELATION

Grinold's Fundamental Law of Active Management states that a manager's IR is a function of both a manager's skill in selecting attractive investments, and the breadth of independent investments from which he can draw. Breadth is related to the number of independent sources of return available in a manager's investment universe, and the number of times the manager turns over the portfolio. A manager who executes a strategy on small but diverse universe with high turnover may equate, in terms of breadth, with a manager who executes on a large but homogenous universe with low turnover. Grinold defines the IR as

$$IR = skill \times \sqrt{breadth}$$

or

$$IR = IC \times \sqrt{N \times T}$$

where IC represents the Information Coefficient, measured as the correlation between forecasts and outcomes for each bet, N is the number of independent bets available in the manager's eligible universe, and T is the number of times each bet is evaluated in the measurement period.

For the balance of this paper we will make the simplifying assumption that managers who focus on different segments of the market have equivalent skill and equivalent turnover in order to isolate the impact of breadth on the Information Ratio (IR). This is somewhat intuitive because there is no reason to believe that managers operating in one universe, say stock selection in a specific market, are any more or less skilled than managers operating in a different universe, such as asset allocation. Furthermore, managers exert some control over their eligible investment universe, and substantial control over turnover in many cases, suggesting these dimensions might be more appropriately classified as control variables.

In his original paper, Grinold equates the number of independent bets with the number of securities in a manager's investable universe. For example, a large-cap equity manager may be restricted to investing in securities in the S&P 500, so he has 500 securities to choose from each time he makes a 'bet'. In contrast, a tactically oriented investor who focuses on asset allocation may choose between, say, 10 major asset classes. By extension from Grinold's interpretation, and assuming similar skill and turnover for both managers, the tactical manager would be expected to have an IR of

$$\frac{\sqrt{500}}{\sqrt{10}} = 7.07$$

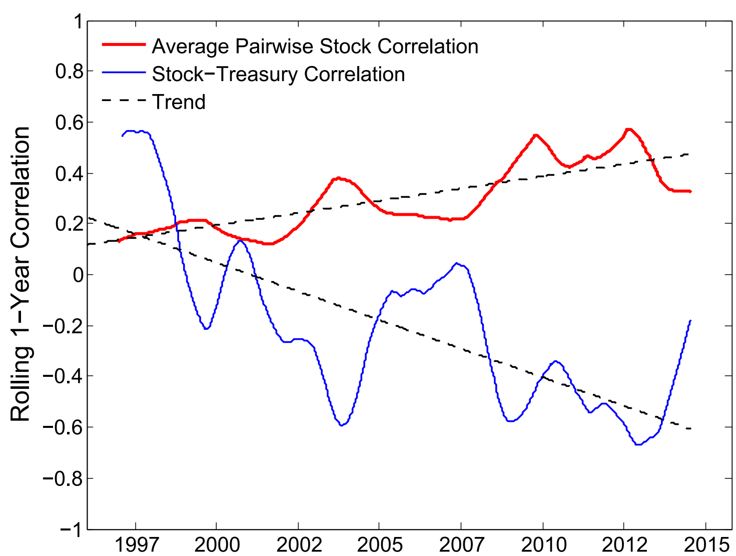
of the IR of the large cap equity manager. For the purpose of consistent nomenclature, we will call this ratio the proportional Information Ratio Multiple (IRM) for the balance of the paper. Thus, the IRM will quantify the square root of the number of independent bets in an investment universe, and it is the magnitude of this factor that impacts relative Information Ratios.

Unfortunately, Grinold's interpretation makes the strong assumption that the 500 securities in the large-cap stock picker's universe, and the asset classes which comprise the tactical manager's universe, represent 'independent bets'. However, this interpretation assumes that the securities in the investable universe are independent - that is, uncorrelated - because if two bets are positively correlated, they are definitionally not independent.

This is easily demonstrated by taking an extreme example of two securities which are 100% correlated. If we equalize their volatilities, clearly holding the two securities in the portfolio would not be any different than holding either one on its own. Of course, stocks in a market are positively correlated, while the correlation between stocks and bonds may be positive or negative at different times. In fact, over the past 20 years, individual stocks have tended toward a pairwise correlation of 0.35 with a 95% range of 0.1 to 0.65, while stocks and Treasuries have averaged a correlation of -0.1 with a 95% range of -.65 to +0.6.

As we will see, the fact that correlations deviate from Grinold's assumption has important implications for managers' potential to generate high Information Ratios (IRs) in security selection versus asset allocation. Furthermore, if the trend over the past 20 years continues, stock correlations may continue to rise while stock and bond correlations decline, per Figure 1, with important implications for trends in marginal breadth.

Figure 1: Average Pairwise 252 Day Rolling Correlations for S&P500 Stocks and for S&P50 vs. U.S. Treasuries.



PRINCIPAL COMPONENT ANALYSIS: A PRIMER

Principal Component Analysis (PCA) is a method of extracting the endogenous, or latent, dynamics that exist in a system. For our purpose, we will be performing PCA on investment universes to determine the independent drivers of portfolio variance. These latent drivers are not readily identifiable like the Fama-French factors (Fama and French, 1993), but are rather embedded in the universe itself. As a result, this method contrasts with typical regression techniques, such as Returns Based Style Analysis or Fama-French factor regressions, which impose exogenous factors on a universe of assets to determine which factors explain portfolio variance.

Principal Component Analysis may be performed on a correlation matrix or a covariance matrix. We will focus primarily on the correlation matrix for reasons that will be made clear below. The process involves transforming the original matrix to derive factor loadings and eigenvalues. The loadings are in the form of a matrix with the same dimensions as the original correlation matrix. For example, if there were thirty assets in the original correlation matrix, the loadings matrix will also have 30 rows and 30 columns. Each column of the loadings matrix represents a latent factor that explains a portion of the movement of the underlying portfolio. Each row corresponds to an asset contained in the original investment universe. By design, each factor is independent of the other factors.

When PCA is applied to portfolio analysis, columns in the loadings matrix are referred to as principal portfolios because they are composed of long or short positions in the constituent assets. The returns from these principal portfolios can be observed by applying the weight vector to the asset returns, and, because they are independent, the returns to each principal portfolio will have a correlation of exactly 0 to one another.

Each principal portfolio has a corresponding eigenvalue, which describes the proportion of total portfolio standardized variance attributable to that factor. When the correlation matrix is used in the analysis, the sum of standardized variances is equal to the number of variables or assets in the universe under analysis; when the covariance matrix is used the eigenvalues sum to the total portfolio variance. Figure 2 plots the eigenvalues or standardized variances derived from a PCA of the correlation matrix formed from the 30 stocks in the Dow Jones over the 500 days ending June 0th, 2014, while Figure 3 shows the principal portfolios of constituent stocks, which load positively or negatively on each factor.

In examining the standardized variances in Figure 2, we note that the first factor (Component 1) exerts a disproportionate impact on the Dow Jones 30 stock portfolio. In fact, it is responsible for about 6× the variance of the next most explanatory factor, and 35% of total portfolio standardized variance. Generally, in the case of an equity-only universe, the factor that captures the most portfolio

Figure 2: Factor variances for Dow 30 stocks derived from 500 day correlation matrix through June 30, 2014.

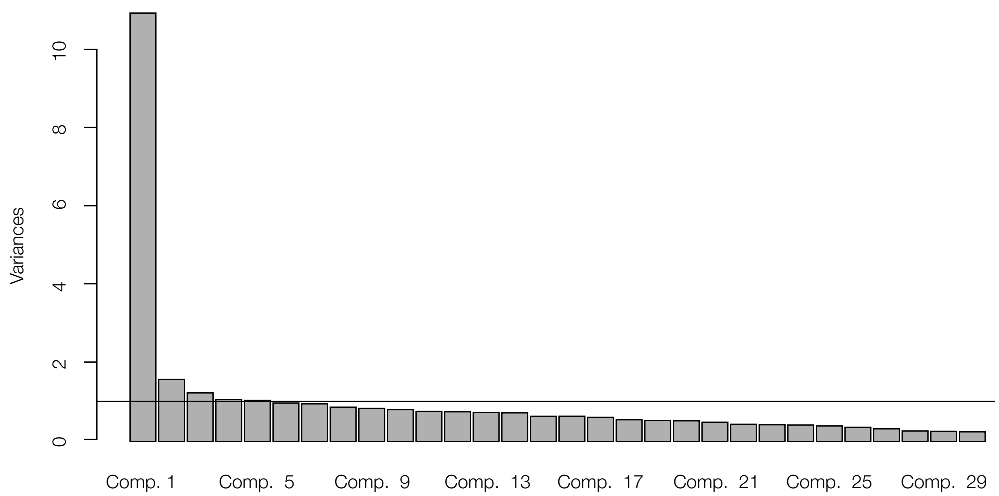


Figure 3: Loading sign heat map for Dow 30 stocks derived from 500 day correlation matrix through June 30, 2014.

AXP	0.22	- 0.01	0.10	- 0.10	- 0.03	- 0.02	0.16	- 0.14	- 0.06	0.16
BA	0.19	- 0.03	0.18	- 0.18	- 0.14	0.12	0.58	- 0.11	- 0.45	- 0.43
CAT	0.20	- 0.06	0.13	- 0.09	0.01	0.07	0.20	0.08	0.66	0.01
CSCO	0.19	- 0.09	0.11	- 0.67	- 0.52	- 0.34	- 0.09	0.24	- 0.11	- 0.01
CVX	0.17	- 0.04	0.07	- 0.04	- 0.03	- 0.02	0.00	0.14	0.08	0.03
DD	0.20	- 0.07	0.08	- 0.03	- 0.05	0.00	0.13	0.00	0.15	0.03
DIS	0.21	- 0.02	0.08	- 0.13	- 0.02	0.02	0.01	- 0.21	- 0.15	- 0.32
GE	0.21	- 0.05	0.09	- 0.06	- 0.07	- 0.02	0.00	0.06	0.22	- 0.01
HD	0.16	- 0.02	0.05	- 0.13	- 0.01	0.01	- 0.02	- 0.20	- 0.08	0.45
IBM	0.16	- 0.04	0.09	- 0.14	- 0.08	0.14	- 0.20	- 0.17	0.00	- 0.21
INTC	0.19	- 0.07	0.10	- 0.33	- 0.42	- 0.14	0.17	0.61	- 0.23	0.03
JNJ	0.14	- 0.04	0.05	- 0.14	- 0.04	- 0.10	- 0.08	- 0.02	- 0.08	0.03
JPM	0.23	- 0.04	0.12	- 0.06	- 0.04	0.17	0.03	0.11	0.15	- 0.02
KO	0.28	- 0.43	-0.85	- 0.04	- 0.01	0.05	0.03	0.01	0.01	0.00
MCD	0.11	- 0.01	0.01	- 0.03	- 0.01	- 0.01	- 0.08	0.00	0.05	- 0.03
MMM	0.19	- 0.04	0.09	- 0.03	- 0.02	- 0.04	0.10	0.01	0.03	0.05
MRK	0.14	- 0.03	0.05	- 0.12	- 0.01	- 0.29	- 0.16	0.25	- 0.19	0.11
MSFT	0.19	- 0.04	0.11	- 0.40	- 0.53	- 0.43	- 0.12	- 0.37	- 0.13	0.07
NKE	0.35	- 0.88	-0.32	- 0.06	- 0.01	- 0.03	0.00	0.02	0.02	- 0.02
PFE	0.16	- 0.06	0.05	- 0.17	- 0.04	- 0.23	- 0.15	0.13	- 0.17	0.19
PG	0.13	- 0.04	0.02	- 0.13	- 0.01	- 0.14	- 0.24	- 0.09	- 0.05	0.03
T	0.15	- 0.05	0.06	- 0.08	- 0.19	- 0.25	- 0.22	- 0.13	- 0.11	- 0.38
TRV	0.15	- 0.04	0.06	- 0.12	- 0.06	0.01	- 0.02	- 0.05	- 0.12	0.04
UNH	0.16	- 0.02	0.08	- 0.08	- 0.33	0.55	- 0.45	0.25	- 0.17	0.16
UTX	0.21	- 0.04	0.09	- 0.06	- 0.09	0.07	0.20	- 0.07	0.02	- 0.02
VZ	0.14	- 0.04	0.04	- 0.14	- 0.27	0.26	- 0.20	- 0.25	0.06	- 0.46
WMT	0.11	- 0.01	0.02	- 0.17	- 0.07	0.02	- 0.14	- 0.08	- 0.04	0.04
XOM	0.16	- 0.05	0.08	- 0.05	- 0.01	0.00	0.00	0.08	0.10	0.01

variance is considered to be market beta, and captures the degree to which the movement of the average stock is related to the movement of the market as a whole. This is validated by the observation that every stock in the Dow loads positively on Component 1. Other factors might capture sector groupings, or sensitivity to interest rates, or arbitrage relationships, though in practice it is difficult to link a latent factor derived through PCA to more traditional grouping structures.

Since principal portfolios are definitionally independent sources of risk, we can use them to determine the number of independent bets. That being said, correlations are noisy, therefore it is necessary to determine which principal portfolios are statistically significant.

Guttman (1954) and Kaiser (1960, 1970), asserted that in order to be significant, “a factor must account for at least as much variance as an individual variable” (Nunnally and Bernstein, 1994). According to the Kaiser-Guttman method, since the average of all standardized eigenvalues is 1, only factors with standardized eigenvalues greater than 1 can be considered to be significant.

Note that in Figure 2 we have added a horizontal line at 1 demarcating the cutoff threshold for significance. From visual inspection it's clear that 5 factors exceed the threshold, so on this basis we might say that there are 5 significant independent factors at play in this asset universe over the time period analysed. However, as we will see below, all is not as it seems.

ASSET ALLOCATION AND SECURITY SELECTION

Staub and Singer provide a convincing argument for the use of an estimated correlation matrix as the measure of portfolio risk, in place of the more commonly used covariance matrix. They argue that asset risk, which is typically measured by return variance, can be rescaled without affecting inter-asset correlations. That is, individual asset variance can conform to unitary variance through the use of leverage. It follows then that any information regarding the risk of a portfolio of assets resides not in the magnitude of risk, but rather in the directions of risk, which is captured by the correlation matrix.

The authors set out to see what proportion of total global breadth is attributable to asset allocation relative to security selection. To do so, they created a large correlation matrix to capture the correlation structure among the following levels of grouping:

1. The investment decision: invest in risky assets versus holding cash
2. Asset classes: stocks or bonds
3. Geographic markets within each asset class
4. Securities within each geographic market

In addition they assume that:

- There are 20 independent stock markets and 20 independent bond markets
- Each independent market is composed of 100 securities

Note that the decision to invest in risky assets versus cash invokes a decision about what mix of asset classes to hold (in this case, proportion of stocks and bonds). Once that proportion is chosen, the investor must choose which geographic markets to own, and once that decision is made what individual stocks and/or bonds to hold in those markets. In this way, each incremental layer of portfolio decision has a cascading impact on more granular sets of assets down the chain.

Each level of grouping contains correlation information about the levels above and below. For example, owning stocks anywhere in the world means you are correlated with the general risk of owning risky assets; more highly correlated yet with the risk of global equities, and; most highly correlated with the geographic equity market the stock is listed on. As such, the authors assume the following:

- Individual stocks in a market have a correlation of 0.5
- Individual bonds in a market have a correlation of 0.8
- Stock indexes of different national markets have a correlation of 0.4
- Bond indexes of different national markets have a correlation of 0.6
- Stock indexes and bond indexes of the same market have a correlation of 0.3
- Stock indexes and bond indexes of different national markets have a correlation of 0.2

In a general sense, this decision tree describes a significant portion of the opportunity set for most large institutions, and exceeds the opportunity set of most private investors.

With these assumptions in place, the authors construct a correlation matrix of dimension 4000 × 4000 (100 stocks × 20 stock indexes + 100 bonds × 20 bond indexes) to quantify all of the inter- and intra-market relationships. They then apply Principal Component Analysis to identify the proportion of independent portfolio breadth attributable to each level of grouping.

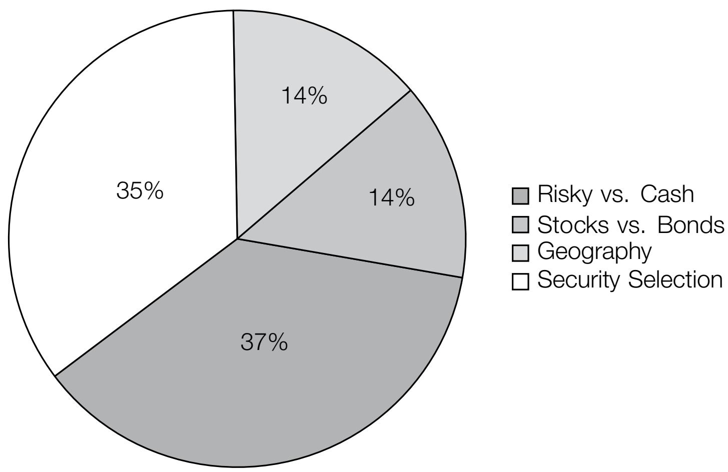
While in practice it is difficult to confidently link latent factors from PCA to real-world factors, a properly constructed analysis, such as Staub and Singer's, lends itself more concretely to traditional groupings. For example, in Staub and Singer's analysis of multiple groupings of assets and stocks, the second factor loads with opposite signs on equities and bonds, and thus by logical extension can be viewed as the asset class (equity versus bonds) decision factor. Given this embedded structure, the authors have confidence in labelling the factor portfolios by the traditional groupings described above, and thus derive the proportion of portfolio variance attributable to each grouping by the cumulative amount of standardized variance for all factors up to, and including, that level.

For example, Factor 1 explains 37% of standardized variance, and captures the risky assets versus cash decision, while Factor 2, which captures the broad stock versus bond decision, explains a further 14%, so that a total of 51% of total standardized variance is explained by these two decisions alone. Adding the 40 factors related to geographic allocation explains another 14%, bringing the total proportion of explained variance to 65%. The proportion of total portfolio breadth attributable to each level of grouping is presented in Figure 4.

In this way we can determine that, with the authors correlation assumptions, 65% of portfolio variance is encapsulated by a combination of the investment versus cash decision; the asset allocation decision; and the market selection decision. These decisions are all the domain of Tactical Alpha. The remaining 35% is what is available for individual security selection decisions in each of the individual stock and bond markets.

While this particular analysis does not specifically quantify the number of independent bets derived from each grouping, it does imply that the domain of Tactical Alpha represents a majority of the total standardized breadth available to an unconstrained global portfolio manager. Taking it one step further with Grinold's equations, we might also assert that investors who ignore Tactical Alpha leave much of the available Information Ratio on the table because a manager who simply

Figure 4: Proportion of total portfolio breadth attributable to each level of grouping.



focuses on security selection in one market has fewer truly independent sources of risk to choose from. We will perform an explicit analysis of independent bets below.

ATTRIBUTION WITH DYNAMIC CORRELATIONS

Staub and Singer made the simplifying assumption that asset classes and securities have stable correlations, but Figure 1 clearly demonstrates that correlations between individual securities in a market, and between stocks and bonds, change materially over time. Given the dynamic nature of correlations, we were keen to measure how portfolio variance attribution shifts between Tactical Alpha decisions and security selection decisions under different correlation regimes.

As such, we performed the same analysis as Staub and Singer for combinations of domestic stock/bond correlations between -1 and +1, and average pairwise security correlations in the same domestic market between 0 and 1 (because average security correlations in a single market rarely, if ever, drop below zero). Note that we held all other correlations between the stock and bond markets of different geographies to the same values assumed by Staub and Singer.

We performed 20 stock/bond correlations × 20 pairwise stock correlations = 400 combinations of the Staub and Singer analysis. The results are shown in Figure 5, where each cell in the matrix quantifies the proportion of total portfolio information that is available for Tactical Alpha managers to exploit given each combination of stock/bond and pairwise stock correlations. The remainder is what is available for traditional security selection managers to exploit.

Figure 5: Proportion of standardized variance attributable to Tactical Alpha decisions at various correlations.

Correlation between domestic stocks	Correlation between domestic stocks and bonds																				
	-1	-0.9	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.00	0.64	0.63	0.62	0.61	0.59	0.58	0.57	0.55	0.54	0.53	0.51	0.51	0.50	0.51	0.51	0.53	0.54	0.55	0.57	0.58	0.59
0.05	0.65	0.64	0.63	0.62	0.61	0.59	0.58	0.57	0.55	0.54	0.53	0.52	0.51	0.52	0.53	0.54	0.55	0.57	0.58	0.59	0.61
0.10	0.66	0.65	0.64	0.63	0.62	0.61	0.59	0.58	0.57	0.55	0.54	0.53	0.53	0.53	0.54	0.55	0.57	0.58	0.59	0.61	0.62
0.15	0.68	0.67	0.66	0.64	0.63	0.62	0.61	0.59	0.58	0.57	0.55	0.54	0.54	0.54	0.55	0.57	0.58	0.59	0.61	0.62	0.63
0.20	0.69	0.68	0.67	0.66	0.65	0.63	0.62	0.61	0.59	0.58	0.57	0.55	0.55	0.55	0.57	0.58	0.59	0.61	0.62	0.63	0.65
0.25	0.70	0.69	0.68	0.67	0.66	0.65	0.64	0.62	0.61	0.59	0.58	0.57	0.56	0.57	0.58	0.59	0.61	0.62	0.64	0.65	0.66
0.30	0.71	0.71	0.70	0.69	0.67	0.66	0.65	0.64	0.62	0.61	0.60	0.58	0.58	0.58	0.60	0.61	0.62	0.64	0.65	0.66	0.67
0.35	0.73	0.72	0.71	0.70	0.69	0.68	0.67	0.65	0.64	0.63	0.61	0.60	0.59	0.60	0.61	0.63	0.64	0.65	0.67	0.68	0.69
0.40	0.74	0.73	0.72	0.72	0.71	0.69	0.68	0.67	0.66	0.64	0.63	0.61	0.60	0.61	0.63	0.64	0.66	0.67	0.68	0.69	0.71
0.45	0.76	0.75	0.74	0.73	0.72	0.71	0.70	0.69	0.67	0.66	0.65	0.63	0.63	0.63	0.65	0.66	0.67	0.69	0.70	0.71	0.72
0.50	0.77	0.76	0.75	0.75	0.74	0.73	0.72	0.70	0.69	0.68	0.66	0.65	0.65	0.65	0.66	0.68	0.69	0.70	0.72	0.73	0.74
0.55	0.78	0.78	0.77	0.76	0.75	0.74	0.73	0.72	0.71	0.70	0.68	0.68	0.68	0.68	0.68	0.70	0.71	0.72	0.73	0.74	0.75
0.60	0.80	0.79	0.79	0.78	0.77	0.76	0.75	0.74	0.73	0.72	0.70	0.70	0.70	0.70	0.70	0.72	0.73	0.74	0.75	0.76	0.77
0.65	0.81	0.81	0.80	0.79	0.79	0.78	0.77	0.76	0.75	0.74	0.73	0.73	0.73	0.73	0.73	0.74	0.75	0.76	0.77	0.78	0.79
0.70	0.83	0.82	0.82	0.81	0.80	0.80	0.79	0.78	0.77	0.76	0.75	0.75	0.75	0.75	0.75	0.76	0.77	0.78	0.79	0.80	0.80
0.75	0.85	0.84	0.84	0.83	0.82	0.82	0.81	0.80	0.79	0.78	0.78	0.78	0.78	0.78	0.78	0.78	0.79	0.80	0.81	0.82	0.82
0.80	0.86	0.86	0.85	0.85	0.84	0.83	0.83	0.82	0.81	0.80	0.80	0.80	0.80	0.80	0.80	0.80	0.81	0.82	0.83	0.83	0.84
0.85	0.88	0.87	0.87	0.87	0.86	0.85	0.85	0.84	0.84	0.83	0.83	0.83	0.83	0.83	0.83	0.83	0.84	0.84	0.85	0.85	0.86
0.90	0.90	0.89	0.89	0.88	0.88	0.87	0.87	0.86	0.86	0.85	0.85	0.85	0.85	0.85	0.85	0.85	0.86	0.86	0.87	0.87	0.88
0.95	0.91	0.91	0.91	0.90	0.90	0.89	0.89	0.89	0.88	0.88	0.88	0.88	0.88	0.88	0.88	0.88	0.88	0.89	0.89	0.89	0.90
1.00	0.93	0.93	0.92	0.92	0.92	0.92	0.91	0.91	0.90	0.90	0.90	0.90	0.90	0.90	0.90	0.90	0.90	0.91	0.91	0.92	0.92

Some examples might help tease out the salient information from Figure 5. First off, the black borders around the vertical column at -0.1 highlight the average correlation between the S&P 500 and U.S. Intermediate Treasuries over the past 20+ year period from Figure 1. The horizontal black borders highlight the average value for pairwise U.S. individual stock correlations over the same period. Where they intersect observe a value of 0.63 or 63%, which is the proportion of total variance attributable to the asset allocation decision under those correlation assumptions. Note then that the amount available from security selection is simply 1-0.63, or 37%. So under average conditions, most of the available breadth is derived from Tactical Alpha decisions, not security selection decisions.

A few other values are highlighted because they are of particular interest. First, the red bordered value of 0.65 corresponds to the assumptions used by Staub and Singer of correlations of 0.5 between domestic stocks, and 0.3 between domestic stock indexes and domestic bond indexes. Note that our analysis confirms their conclusions. In addition, the blue bordered value of 0.57 represents the intersection of the 95th percentile stock/bond correlation (when stocks and bonds are highly correlated) and the 5th percentile average pairwise stock correlation (when individual stocks are not highly correlated), which identifies the most favourable times for stock pickers. Incredibly, even at peak times for stock pickers they still have at their disposal less than half of the opportunity set that would be available to them if they expanded their scope into Tactical Alpha.

Lastly, we highlight the proportional breadth during periods of market stress, when pairwise stock correlations have historically converged towards 1, and stock bond correlations have historically declined. As such, the green bordered value represents the intersection of 95th percentile average pairwise stock correlation with the 5th percentile stock / bond correlation, a period which clearly favours asset allocation decisions. You can see that at such times the Tactical Alpha decision dominates security selection by almost a factor of 4 to 1.

This goes to show that, while it may be harder to see the value of Tactical Alpha during stable bull markets, under periods of global duress the benefits of Tactical Alpha are prominent.

PART TWO: EMPIRICAL ANALYSIS

DATA

Recall that the goal of this paper is to quantify the opportunity to improve portfolio Information Ratios by prioritizing Tactical Alpha relative to traditional security selection. As discussed in Part 1, we confine the domain of Tactical Alpha to include decisions related to overall portfolio exposure to risky assets versus cash; the asset allocation decision (bonds, stocks, real estate, commodities, etc.), and; geographic allocation (Japanese stocks, U.S. onds, etc.).

We require data for cash, broad asset classes, several geographic regions, and individual securities. For cash we use the 90-day US T-Bill total return, sourced from Global Financial Data. We chose asset classes which are representative of approximately 75% of global market capitalization, and that have liquid, U.S. listed Exchange Trade Fund tracking securities. Dividend adjusted total returns to ETF proxies were used from inception, and extended using gross total returns from underlying indexes from MSCI, S&P, FTSE, Barclays, Deutsche Bank, and Dow Jones back to January 1, 1995.

Consistent with Staub and Singer, we are interested in the marginal increase in portfolio breadth that results from adding individual securities to the asset class universe. We used a medium sized universe consisting of the S&P 100 index, and a larger universe of S&P 500 stocks. Our analysis extends back to January 1, 1995, so we sourced data for the current constituents of these indexes

Table 2: Asset class universe.

ETF Proxy	Index
VTI	U.S. Total Market Index
VGK	FTSE Europe Index
EWJ	MSCI Japan Index
EEM	MSCI Emerging Markets Index
IYR	Dow Jones US REIT Index
RWX	Dow Jones Global ex-U.S. Select Real Estate Securities Index
IEF	Barclays US 7-10 Year Intermediate Treasury Bond Index
TLT	Barclays 20+ Year Treasury Bond Index
DBC	DBIQ Optimum Yield Diversied Commodity Index
GLD	Spot Gold
IRX	U.S. 90-Day Treasury Bond [Cash]

with histories that extend back to that date. This yields 81 names for the S&P 100 universe and 357 names for the S&P 500 universe. We used daily total return data adjusted for dividends, and sourced from Bloomberg.

EMPIRICAL EXAMPLE 1: INDEPENDENT BETS FROM RANDOM MATRICES

The attribution analysis by Staub and Singer is obviously a simplification of reality. The authors used fixed artificial correlation matrices in order to facilitate a meaningful understanding of the principal portfolios derived from Principal Component Analysis (PCA). In reality, historical returns are noisy and result in correlation matrices that are far from fixed, and thus lack the explicit structure of the Staub and Singer analysis. Consequently, it is challenging to dynamically attribute financial meaning to the principal factors derived from real asset returns, rendering the analysis described by Staub and Singer infeasible with real data.

In addition, while directly related, Staub and Singer's analysis did not have the explicit objective of measuring the true number of independent bets in any segment of their universe of assets. Since it is the number of independent bets - or more specifically the square root of the number of bets - that impacts the IRM in the context of the Fundamental Law of Active Management, this will be the focus of the balance of our analysis.

As noted above, it is a simple matter to extend the PCA framework to determine the number of significant independent bets in an asset universe. The foundation for this analysis was described by Buckle (2004), who quantified in some detail the impact of asset correlation on breadth. Buckle concluded that breadth declines in response to rising average pairwise correlations, and that the acceleration of decay is greater for larger universes. In other words, from an Information Ratio (IR) standpoint, a manager who invests in a large universe of correlated assets may be at a disadvantage relative to a manager who acts on a small universe of uncorrelated assets.

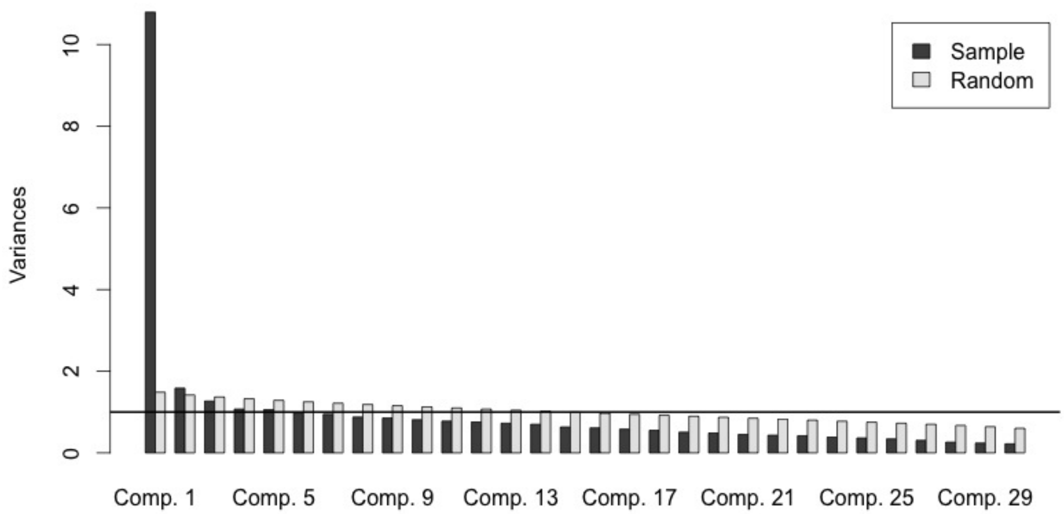
Following the work of Buckle, Polakow and Gebbie (2008) applied Singular Value Decomposition, a close cousin of PCA, to derive the number of independent bets on the South African Stock Exchange. They used the Kaiser-Guttman method to eliminate factors which exhibit standardized variance less than 1, for reasons explained above.

Horn (1965) observed that randomly generated correlation matrices of the same dimension and character will present with what appear to be significant factors, according to the KaiserGuttman

method, purely by chance. As such, they showed that the Kaiser-Guttman method systematically overestimates the number of significant factors, and proposed that factors should only be considered significant if they explain a greater proportion of variance than what would be expected from random chance. Zwick and Velicer (1986) also reviewed the Kaiser-Guttman method and validated Horn's conclusions.

Figure 6 shows factor variances derived from a correlation matrix of Dow 30 stocks over the 500 days ending June 30, 2014 compared with factor variances derived from 1000 random matrices of the same dimension and character as the sample matrix. Note that Component 1 substantially exceeds the threshold of what would be expected from random matrices, while component 2 marginally exceeds the threshold. However, factors 3 through 30 fail to exceed the threshold, suggesting these factors are not statistically significant. It follows that over the past 500 days the Dow Jones index of 30 stocks represents only 2 statistically significant independent bets.

Figure 6: Number of eective bets obtained from Dow Jones 30 stocks vs. random matrices of similar dimension.



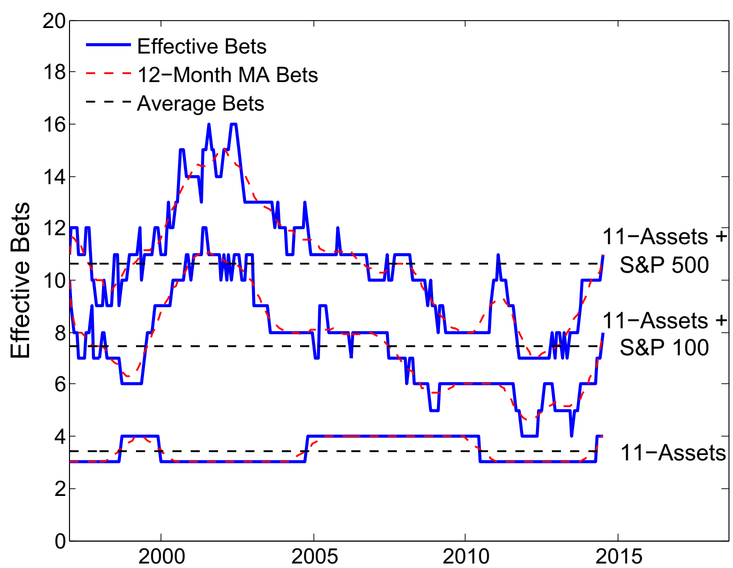
Recall that we are interested in how the number of independent bets, or breadth, of an asset universe contributes to the IR of an active strategy implemented on that universe. Recall also that the IR is proportional to the square root of market breadth. Therefore we are interested in the square root of the number of independent bets, not the raw number, which we have termed the Information Ratio Multiple (IRM) for consistency.

We analyzed the evolution of statistically significant independent bets through time using random matrix thresholds for the following universes:

- A universe consisting of the 11 asset classes in Table 2.
- A universe consisting of the same 11 asset classes plus 81 constituents of the S&P 100 with histories back to 1995.
- A universe consisting of the 11 asset classes plus 357 constituents of the S&P 500 with histories back to 1995.

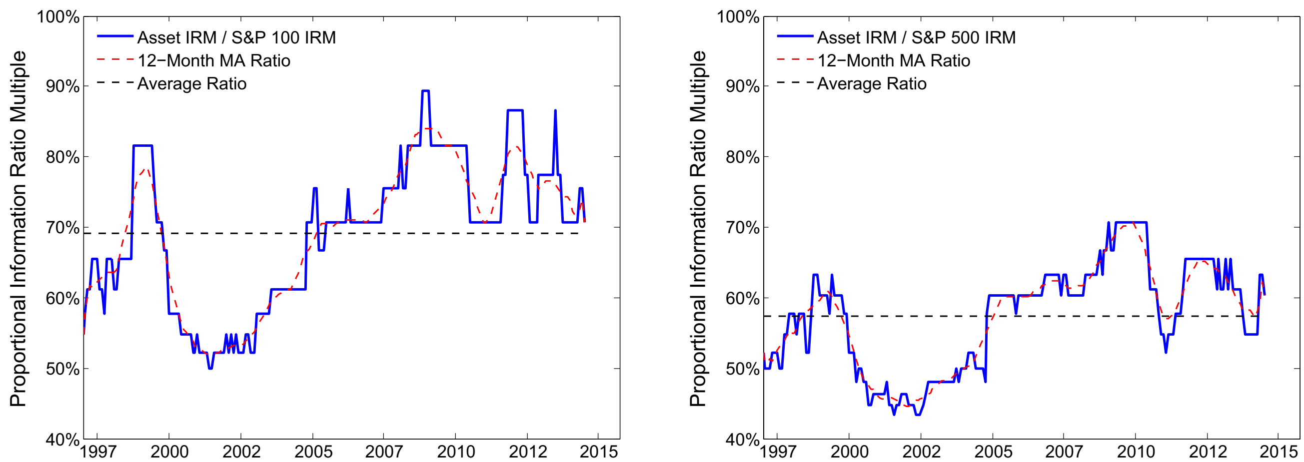
The analysis was conducted from 1995 through June 2014 using a rolling 500 day correlation matrix. Figure 7 describes the evolution of the number of statistically significant bets for each universe through time. By visual inspection, we can see that the asset class universe renders between 3 and 4 independent bets over time, while the S&P 100 universe yields 6 to 12 bets and the S&P 500 universe generates 7 to 16 bets.

Figure 7: Comparison of the number of eective bets obtained from random matrices.



If we were interested in the raw number of bets, we might conclude that the opportunity to expand IRs is an order of magnitude larger for universes which include a large number of stocks. However, we are interested in the square root of these bets, which alters the conclusion significantly. Figure 8 compares the proportion of IRM that is attributable to asset allocation by computing the rolling ratio of Asset IRM to both Assets + S&P 100 IRM and Assets + S&P 500 IRM.

Figure 8: Evolution of the proportional attribution of the breadth term in Grinold's Fundamental Law of Asset Management related to asset allocation vs. security selection.



It is now a simple matter to identify the relative contribution of each universe to total portfolio Information Ratio. The average of the square root of breadth for the asset class universe is 1.84 versus 2.71 for the universe containing asset classes and the S&P 100 stocks, for a ratio of 1.84/2.71 or 68%. The average of the square root of breadth for the 357 stock modified S&P 500 universe is 3.25, for a ratio of 1.84/3.25 or 57%. Therefore we can conclude that the asset class universe contributes about 68% of breadth when we introduce S&P 100 stocks, and 57% when we introduce S&P 500 stocks.

EMPIRICAL EXAMPLE 2: SIMULATED INVESTORS

The results above provide strong validation of the breadth opportunity available from asset allocation relative to security selection, but it does not address the impact of these decisions in practice. As such, we performed a normative analysis to illustrate the boost in IR which can be derived from Tactical Alpha.

Nguyen (2004), channeling Samuelson, asserted that Tactical Alpha is often overlooked by active investors, resulting in an implicit increase in the existence of inter-market inefficiencies. Such inefficiencies can deliver attractive excess returns and are therefore significant sources of alpha.

Nguyen identified that Tactical Alpha is often dismissed because it is framed as 'market timing' and is generally considered in the context of tactical shifts between stocks and bonds in a domestic market. However, this is inconsistent with most modern applications of this practice, which incorporate several sources of asset class, geographic and factor risks. Nguyen recognized that, for the same level of manager skill, IR increases for an investor who considers a broad asset class universe instead of just two asset classes. To justify this thesis, Nguyen performed an experiment comparing the likelihood of two equally skilled investors in achieving excess alpha, given similar active risk.

Specifically, she examined the distribution of Information Ratio distributions representing two investors:

- Investor 1: who can choose between two asset classes, namely U.S. stocks versus U.S. bonds, consistent with the traditional definition of 'market timing'.
- Investor 2: who can choose between 18 global equity markets and 10 global bond markets simultaneously (28 asset classes in total).

For each investor, she performed 75 randomized simulations resulting in 75 randomized equity paths. Each portfolio was rebalanced monthly, with a 55% chance of correctly choosing assets with positive monthly returns, and assumed the same amount of active risk.

Per Nguyen, the average simulated outcome revealed that the investor who considers 28 global assets achieves a median annualized alpha of 2.8% and an IR of 1.3. This compares favourably with the 2 asset investor, which achieves a median annualized alpha of 0.7% and an IR of 0.3, it is clear that it is easier to achieve higher alphas and IRs with larger asset class universes.

With this framework in mind, we extended the analysis one step further. We now consider three equally skilled investors:

• Investor 1: who can choose any portfolio of 2 assets from our 11 asset class universe.
• Investor 2: who can choose any portfolio of any number of assets from our 11 asset class universe.
• Investor 3: who can choose any portfolio of any number of assets from our 11 asset class universe and the S&P 100.

For each investor, we conducted 1000 randomized simulations, resulting in 1000 randomized equity paths. As before, each investor rebalanced their portfolios monthly, had a 55% chance of correctly choosing assets or stocks with positive monthly returns, and targeted similar active risk. In order to eliminate bias which might have been introduced based from the fact that any one asset class exhibited higher risk adjusted returns over our sample period, we demeaned the returns of both the asset universe and S&P 100 such that the long-term geometric mean of each asset and stock was zero.

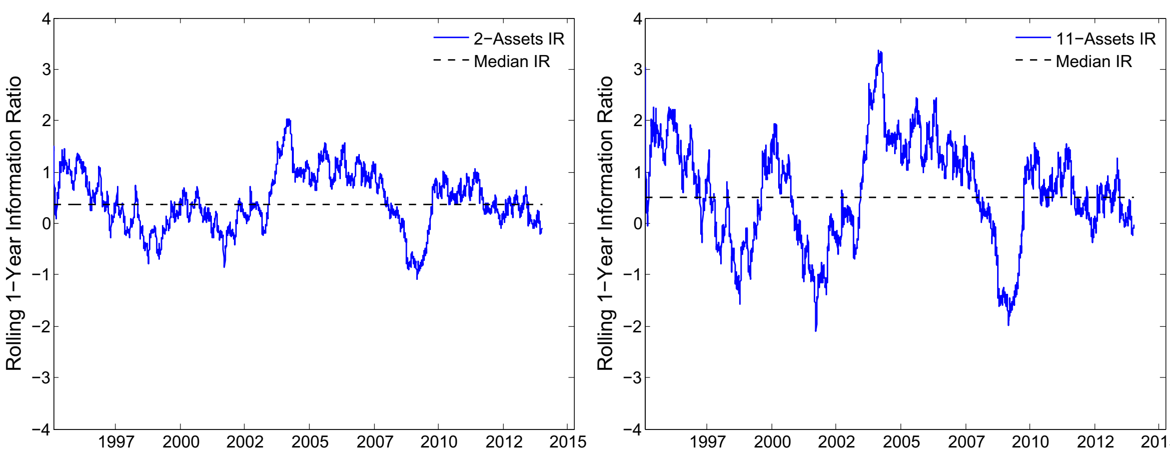
We computed the annualized IR over the entire testing period for each investor's 1000 simulations. Lacking an obvious benchmark for the IR computation, we simply use the riskfree rate, which in a demeaned universe is 0%. The 25th, 50th and 75th percentile IR outcomes are presented in Table 3. Observe that the median IR for the long-only 11 asset universe (Investor 2) is 0.51 whereas the median IR for the asset plus stock universe (Investor 3) is 0.72. This indicates that, all else equal, investors accrue 71% of the benefits of investing in the Assets + S&P 100 universe by solely investing in the 11-asset class universe.

In Figure 9 we present the rolling annual average IR for the three investors. Observe that there are extensive periods, such as in 2000, 2002-2008 and 2011, where the IR derived from the 10-asset universe and cash exceeds the IR derived from the Assets + S&P 100 universe. This suggests that in many cases stock selection adds very little value to the portfolio, and can in fact dilute the information gathered through asset allocation alone by introducing noise or over-concentration in certain bets.

Table 6: Information Ratios of three simulated investors.

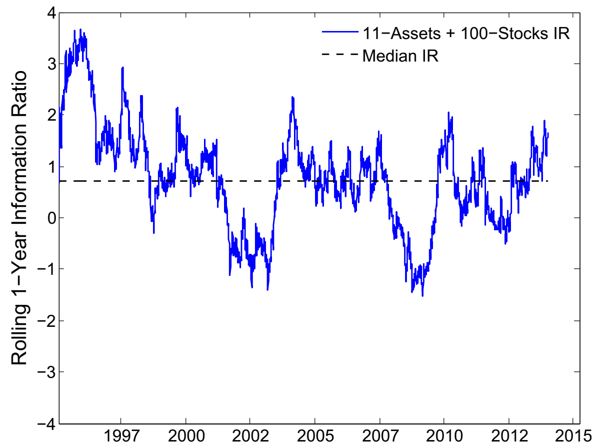
Information Ratio	Investor 1	Investor 2	Investor 3
25%-ile	0.244	0.446	0.696
50%-ile	0.380	0.513	0.724
75%-ile	0.519	0.578	0.753

Figure 13: Rolling 1-Year Average Information Ratio for Simulated Investors.



(a) Investor 1

(b) Investor 2



(c) Investor 3

CONCLUSION

We set out to explore the opportunity for investors to achieve higher Information Ratios by expanding the domain of active management into Tactical Alpha. Previous research from Brinson, Ibbotson and Kaplan laid a strong empirical foundation for the role of asset allocation in the returns of institutional portfolios, implying that asset allocation explained 90% of portfolio variance, 40% of cross-sectional institutional portfolio dispersion, and 104% of portfolio returns. Assoe et al. performed constrained simulations to measure the breadth of the opportunity set attributable to asset allocation and security selection, and concluded that, “it cannot be unequivocally declared that one activity is structurally more or less important than the other”.

Staub and Singer asserted that all the meaningful information in a portfolio is contained in the correlation matrix, because holdings can be scaled to contribute any target level of variance. They created a highly structured correlation matrix including linked groupings of asset classes and securities, and used Principal Component Analysis to measure the standardized variance explained by each level of grouping. Their analysis concluded that decisions related to Tactical Alpha explain about 65% of standardized portfolio variance, and proportional breadth, under typical market correlation assumptions.

The Staub and Singer analysis imposed static correlation assumptions, contrary to the noisy correlations observed across assets over time. We invoked the same process as Staub and Singer to study how sensitive breadth attribution is to changes in correlation between stocks and bonds, and among individual stocks. Our analysis suggests that attribution ranges between 0.57 in favour of asset allocation when stocks have weak pairwise correlation while stocks and bonds are strongly correlated (5th and 95th percentiles, respectively); and 0.79 when stocks are strongly correlated and assets are weakly correlated (95th and 5th percentiles, respectively). The average over our observation period yielded an asset class attribution percentage of 0.63, suggesting that about 2/3 of total available breadth is attributable to Tactical Alpha decisions.

In addition, we performed two empirical studies. First we applied PCA with random matrixes to measure the number of statistically significant independent bets in a universe consisting of 10 broad asset classes plus cash, and measured the excess independent bets which accrued from the addition of S&P 100 and then S&P 500 stocks. While the proportion of bets changed through time in response to changes in correlations, on average 2/3 of the Information Ratio Multiple was found to be attributable to Tactical Alpha with the introduction of the S&P 100 universe, while 57% was attributable to Tactical Alpha with the inclusion of the S&P 500 universe.

Lastly, we performed a normative random portfolio simulation based on a framework proposed by Nguyen. Given our assumption, per Grinold, that the IR achieved by equally skilled investors is an increasing function of an investor's portfolio breadth, we consider three equally skilled investors: one who chooses an allocation exclusively between 2 randomly selected assets from the 10 asset universe plus cash; one who chooses any allocation randomly from among the 10 asset universe plus cash, and; one who chooses an allocation from among the 10 assets, cash, and S&P 100 stocks. The median IR from 1000 portfolios formed each month from the asset class plus cash universe was 0.51 versus 0.72 for the universe formed from the assets plus stocks. As a result, we conclude that investors accrued 71% of the benefits from investing in assets plus stocks by investing in assets alone. Furthermore, in examining the proportional attribution through time, we observe that there are extensive periods where stocks do little to bolster overall IRs.

It's clear that investors who dismiss Tactical Alpha as a source of active returns are ignoring a meaningful source of excess risk-adjusted performance. But how might investors go about introducing Tactical Alpha into portfolios? The following resources offer a variety of ways to implement Tactical Alpha as portable alpha overlays or stand-alone strategies:

- Value and Momentum Everywhere (Asness et al., 2013)
- Global Tactical Cross-Asset Allocation: Applying Value and Momentum Across Asset Classes (Blitz and Vilet, 2008)
- Adaptive Asset Allocation: A Primer (Butler et al., 2012)
- Relative Strength Strategies for Investing (Faber, 2010)
- Using a Z-score Approach to Combine Value and Momentum in Tactical Asset Allocation(Wang and Kochard, 2011)
- Modern Tactical Asset Allocation (de Silva, 2006)
- A Factor Approach to Asset Allocation (Clarke et al., 2005)
- Global Tactical Asset Allocation: Exploiting the opportunity of relative movements across asset classes and financial markets (Potjer and Gould, 2007)
- Advanced Theory and Methodology of TAA (Lee, 2000)

REFERENCES

Asness, C.S., Moskowitz, T.J., Pedersen, L.H., 2013. Value and momentum everywhere. The Journal of Finance 68, 929 - 985.

Assoe, K., L'Her, J.F., Plante, J.F., 2006. The relative importance of asset allocation and security selection. The Journal of Portfolio Management 33, 46-55.

Blitz, D., Vilet, P.V., 2008. Global tactical cross-asset allocation: Applying value and momentum across asset classes. Journal of Portfolio Management 1, 23-28.

Brinson, G.P., Hood, L.R., Beebower, G.L., 1986. Determinants of portfolio performance. Financial Analysts Journal 42, 39-44.

Brinson, G.P., Singer, B.D., Beebower, G.L., 1991. Determinants of portfolio performance 2: An update. Financial Analysts Journal 47, 40-48.

Buckle, D., 2004. How to calculate breadth: An evolution of the fundamental law of active portfolio management. Journal of Asset Management 4, 393-405.

Butler, A., Philbrick, M., Gordillo, R., Varadi, D., 2012. Adaptive asset allocation: A primer. Social Science Research Network 1, 1-30.

Clarke, R.G., de Silva, H., Murdock, R., 2005. A factor approach to asset allocation. The Journal of Portfolio Management 32, 10-21.

Faber, M.T., 2010. Relative strength strategies for investing. Social Science Research Network 1, 1-22.

Fama, E.F., French, K.R., 1993. Common risk factors in the returns on stocks and bonds. Journal of Financial Economics 33, 3-56.

Grinold, R.C., 1989. The fundamental law of active management. Journal of Portfolio Management 15, 30-37.

Guttman, L., 1954. Some necessary conditions for common factor analysis. Psycometrika 19, 149-161.

Horn, J.L., 1965. A rationale and test for the number of factors in factor analysis. Psychometrika 30, 179-185.

Ibbotson, R.G., Kaplan, P.D., 2000. Does asset allocation policy explain 40, 90, or 100 percent of performance? Financial Analysts Journal 56, 26-33.

Kaiser, H., 1960. The application of electronic computers to factor analysis. Educational and Psychological Measurement 20, 141-151.

Kaiser, H.F., 1970. A second generation little jiy. Psychometrika 35, 401-415.

Kritzman, M., Page, S., 2003. The hierarchy of investment choice. Journal of Portfolio Management 29, 11-23.

Lee, W., 2000. Advanced Theory and Methodology of Tactical Asset Allocation. Wiley.

Meucci, A., 2010. Managing diversication. Bloomberg Education & Quantitative Research and Education Paper , 74-79.

Nguyen, Q., 2004. The case for global tactical asset allocation. Morgan Stanley Investment Management: Issues of Interest , 1-9.

Nunnally, J., Bernstein, I.H., 1994. Psychometric theory (3rd ed.). New York: McGraw-Hill.

Polakow, D., Gebbie, T., 2008. How many independent bets are there. Journal of Asset Management 9, 278-288.

Potjer, D., Gould, C., 2007. Global Tactical Asset Allocation: Exploiting the opportunity of relative movements across asset classes and nancial markets. Risk Books.

de Silva, H., 2006. Modern tactical asset allocation. CFA Institute Conference Proceedings Quarterly 23, 1-10.

Staub, R., Singer, B., 2011. Asset allocation vs. security selection: Their relative importance.

CFA Institue: Investment Performance Measurement 1, 1-9.

Wang, P., Kochard, L., 2011. Using a z-score approach to combine value and momentum in tactical asset allocation. Social Science Research Network 1, 1-42.

Zwick, W.R., Velicer, W.F., 1986. Comparison of ve rules for determining the number of components to retain. Psychological Bulletin 99, 432-442.